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An M/M/c/N Feedback Queuing System with Encouraged Arrivals, Reverse Reneging and Retention of Reneged Customers

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Abstract: *In this paper authors have modelled current competitive business scenario of firms giving heavy discounts and offers to attract customers and encourage them to join the system to avail service. The impatience rate of such customers is usually very low, which results in huge customer base and may sometimes results in chaotic situation if not managed effectively. Firms who fail to manage the waiting time of customers in the queue may face detrimental consequences in terms of loss of business or goodwill. Often huge customer base puts pressure on the service channels leading to some level of dissatisfaction among certain customers availing service. Such customers may re-join the queue to avail service satisfactorily. Stochastic queuing model developed and solved in this paper for the system facing issues mentioned above can help firms measure their performance well in advance and device strategies for effective management of the system.*

Key words: encouraged arrivals, multi-server queuing model, reverse reneging, feedback customers, retention

1. Introduction

In this competitive business scenario where each firm is striving to sustain, increasing the customer base by introducing various schemes, discounts and offers has become inevitable for the firms to attract customers. Such customers allured by offers and deals towards a particular firm are termed as encouraged arrivals [2]. These customers when join the system are willing to spend more time in the system to avail the offers and their reneging rate is often less. This concept is termed as reverse reneging [17, 18]. After having waited for a time longer than preconceived threshold limit, some customers might get impatient and leave the system without availing service. Losing such customers may prove to be detrimental for the firm. Certain retention strategies can be adopted by the firm to retain these customers to the best possible extent. With huge customer base due to encouraged arrivals and adopted retention strategies, servers experience huge load and sometimes adjust in a way to provide quick service to the customers waiting in the queue. This in turn can lead to some level of dissatisfaction among certain customers. These dissatisfied customers join the queue again for satisfactory service and are termed as feedback customers [15].

This paper is an extension of the work by Authors in [5], incorporating feedback customers and retention of reneged customers. Detailed relevant literature review is presented in [3, 4 and 5]. The purpose of this research is to develop and solve a stochastic queuing model for firms experiencing the contemporary issues of encouraged arrivals, reverse reneging, retention and feedback customers.

2. Model Formulation and Steady-State Solution

Mathematical queueing System is formulated under the basic assumptions as in [5] along with two additional assumptions as follows:

- A dissatisfied customer may rejoin the queue with probability q and may leave the queue satisfactorily with probability $p = 1 – q$.
- The probability of retention of a reneged customer is q' and the probability that customer is not retained, $p' = 1 – q'$.

2.1 Steady – State Mathematical Model

$$0 = -\lambda(1+\eta)P_0 + (\mu p + N\xi p')P_1; \quad n = 0 \tag{1}$$

$$0 = \lambda(1+\eta)P_{n-1} - \{\lambda(1+\eta) + n\mu p + [(N-(n-1))\xi p'\}P_n + \{(n+1)\mu p + (N-n)\xi p'\}P_{n+1}; \; 1 \le n < c \tag{2}$$

$$0 = \lambda(1+\eta)P_{n-1} - \{\lambda(1+\eta) + c\mu p + [(N-(n-1))\xi p'\}P_n + \{c\mu p + (N-n)\xi p'\}P_{n+1}; c \le n \le N-1 \tag{3}$$

$$0 = \lambda(1+\eta)P_{N-1} + (c\mu p + \xi p')P_N; \qquad n = N \tag{4}$$

2.2 Steady – State Solution

Solving (1) – (4) iteratively, we get:

P_n = Probability of n customers in the system

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$$= \begin{cases} \prod_{r=1}^{n} \frac{\lambda(1+\eta)}{[r\mu p + \{N-(r-1)\}\xi p']} P_0, \ 1 \le n \le c; \\ \prod_{r=1}^{c} \frac{\lambda(1+\eta)}{[r\mu p + \{N-(r-1)\}\xi p']} \prod_{s=c+1}^{n} \frac{\lambda(1+\eta)}{[c\mu p + \{N-(s-1)\}\xi p']} P_0, \ c+1 \le n \le N \end{cases} \quad (5)$$

As, $\sum_{n=0}^{N} P_n = 1$, we get

P_o = Probability of no customer in the system

$$= \left[1 + \sum_{n=1}^{c} \left\{\prod_{r=1}^{n} \frac{\lambda(1+\eta)}{[r\mu p+\{N-(r-1)\}\xi p']}\right\} + \sum_{n=c+1}^{N} \left\{\prod_{r=1}^{c} \frac{\lambda(1+\eta)}{[r\mu p+\{N-(r-1)\}\xi p']} \prod_{s=c+1}^{n} \frac{\lambda(1+\eta)}{[c\mu p+\{N-(s-1)\}\xi p']}\right\}\right]^{-1} \quad (6)$$

3. Performance Measures

Having calculated probabilistic measures P_n above, various performance measures, Expected system size $(\boldsymbol{L_s})$, Length of queue $(\boldsymbol{L_q})$, Waiting time of a customer in the queue $(\boldsymbol{W_q})$, Total waiting time of a customer in the system $(\boldsymbol{W_s})$, Average rate of reverse reneging $(\boldsymbol{R_{ren}})$ and Average rate of retention $(\boldsymbol{R_{ret}})$ can be calculated easily by classical queuing approach as follows:

a) $L_s = \sum_{n=0}^{N} nP_n$

b) $W_s = \frac{L_s}{\lambda(1+\eta)}$

c) $W_q = W_s - \frac{1}{c\mu p}$

d) $L_q = \lambda(1+\eta)W_q$

e) $R_{ren} = \sum_{n=1}^{N}\{[N-(n-1)\xi p'\}P_n$

f) $R_{ret} = \sum_{n=1}^{N}\{[N-(n-1)\xi q'\}P_n$

4. Conclusion and Future Work

Model is developed by using classical method for development of queuing model by listing all mutually exclusive cases in form of differential difference equations. The model is then tested by using Laplace transform for validity. After validation, these equations are then converted into steady-state equations as time tends to infinity. Iterative method is used for solving the model. Steady-state probabilistic measures and performance measures are then derived. These results measure overall performance of the system under given conditions. The results are of immense use for any business organization to understand its performance well in advance theoretically. These theoretical results can help firms in designing strategies for managing the system effectively and better service can be provided to the customers, which can help firms gain competitive edge over competitors.

Apart from the results mentioned above many more measures can also be derived by using classical queuing theory approach. The model can be solved for infinite capacity system. Waiting time distribution of the customers can be obtained for the developed model and Cost-Profit analysis can also be performed.

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